$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(v_i - \mu)^2 = \frac{1}{N}\sum_{i=1}^{N}(v_i-\mu)(v_i-\mu)$$

$$= \frac{1}{N}\sum_{i=1}^{N} v_i^2 - 2\mu v_i + \mu^2 = \frac{1}{N}\sum v_i^2 - 2\mu\sum v_i + \sum \mu^2$$

$$= \frac{1}{N}\sum_{i=1}^{N} v_i^2 - 2\mu\sum_{i=1}^{N} v_i + \mu^2 \sum_{i=1}^{N} 1 .$$

now note that $\sum_{i=1}^{N} v_i = N\mu$ & $\sum_{i=1}^{N} 1 = N$

such that $\frac{1}{N}\sum_{i=1}^{N} v_i^2 - 2\mu N\mu + N\mu^2$ is

equivalent. Now,

$$\frac{1}{N}\left[\sum v_i^2 - 2N\mu^2 + N\mu^2\right] = \frac{1}{N}\left(\sum v_i^2 - N\mu^2\right)$$

finally $\frac{1}{N}\sum v_i^2 - \mu^2$ is 1.6.9 on pg 27

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$\sum (x_i - \bar{x})^2 = \sum (x_i - \bar{x})(x_i - \bar{x}) = \sum x_i^2 - \bar{x}x_i - \bar{x}x_i - ,$$

$$\sum x_i^2 - 2x_i\bar{x} + \bar{x}^2 = \sum x_i^2 - \sum x_i\bar{x}2 + \sum \bar{x}^2 =$$

$$\sum x_i^2 - 2\bar{x}\sum x_i + \bar{x}^2 \sum 1$$ = Now the trick.

note that $\sum_{i=1}^{n} 1 = n$ & $\sum x_i = n\bar{x}$

so we have $\sum x_i^2 - 2\bar{x}n\bar{x} + n\bar{x}^2 =$

$$\sum x_i^2 - 2n\bar{x}^2 + n\bar{x}^2 = \sum x_i^2 - n\bar{x}^2$$

now lets get back to full notation.

$S^2 = \frac{1}{n-1}\left[\sum x_i^2 - n\bar{x}^2\right]$ this is close ~~bat~~ but not there. To get to 1.6.14 we need a trick wit $n\bar{x}^2$, So $\bar{x} = \frac{\sum x_i}{n}$ then

$$n\bar{x}^2 = n\left(\frac{\sum x_i}{n}\right)^2 = \frac{n}{n^2}\left(\sum x_i\right)^2 = \frac{1}{n}\left(\sum x_i\right)^2$$

finally $S^2 = \frac{1}{n-1}\left[\sum x_i^2 - \frac{1}{n}\left(\sum x_i\right)^2\right]$